Neutrinos in Particle Physics and Cosmology

- Standard Model (SM) Neutrinos
- Neutrino Oscillations
- Neutrino Masses & Mixings
- Neutrinos in Warped Extra Dimension
- Inflationary Cosmology
- Inflation Models / Leptogenesis
- Summary

UNITS

$$\hbar = c = k = 1$$

$$M_{Planck} \simeq 1.2 \times 10^{19}\,\mathrm{GeV} = G_N^{-1/2}$$

Newton's constant

$$\mathrm{GeV}^{-1} = 2 \times 10^{-14}\,\mathrm{cm}$$

$$\mathrm{GeV} = 1.2 \times 10^{13}\,\mathrm{K}$$

$$\mathrm{Mpc} = 3 \times 10^{24}\,\mathrm{cm} = 1.5 \times 10^{38}\,\mathrm{GeV}^{-1}$$

$$\mathrm{pc} = 3.26\,\mathrm{LY} = 3.1 \times 10^{18}\,\mathrm{cm}$$

Standard Model (SM)

- Gauge Symmetry $\underbrace{SU(3)}_{\text{QCD}} \times \underbrace{SU(2)\times U(1)}_{\text{EW}}$

QCD $\downarrow$ confined phase

EW $\downarrow$ partial higgs phase

- Matter Multiplets

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix},\ u_i^c,\ d_i^c$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix},\ e^c$$

15 fields per family

($i = 1,2,3$ — color index)

No gauge anomalies!

- Higgs $\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \equiv \phi$

$\langle \phi \rangle \neq 0$ breaks $SU(2)\times U(1) \longrightarrow U(1)_{em}$;

Also, charged fermions & $W^\pm$, Z acquire masses.

Predictions (confirmed)

Weak Neutral Currents

Parity Violation in Atoms

$W^{\pm}$, Z gauge bosons

Gluon jets

Asymptotic Freedom

c, t, b quarks; CP violation

Numerous other tests (Stable proton, ..) $\tau_p \gtrsim$ few $\times 10^{33}$ yrs

? Higgs boson ($m_h \gtrsim 114$ GeV)

- The $SU(2)_L \times U(1)_Y$ component of the theory exhibits spontaneous symmetry breaking:

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle H \rangle} U(1)_{em}$$

$$(Q = T_3 + Y)$$

Cf: Superconductor $U(1)_{em} \xrightarrow[\langle \phi \rangle]{} \mathbb{Z}_2$

Cooper pair

The H field is a doublet of $SU(2)_L$ and contains 4 real fields. Three are absorbed by $W^\pm$, Z, while the 4^{th} acquires mass ~ electroweak scale.

EXPT: $m_{h^0} \gtrsim 115$ GeV

- Spontaneous breaking of $SU(2)_L \times U(1)_Y$ provides masses not only for $W^\pm, Z$, but also for quarks & charged leptons:

Yukawa couplings

$$\overline{\begin{pmatrix} t \\ b \end{pmatrix}_L} \; t_R \; H \quad , \text{etc.}$$

With no 'right-handed' neutrinos, we (naively) expect the observed neutrinos to be 'massless'.

However, this is not quite true!

Consider the combination

$$\nu_L H \nu_L H$$ ← violates lepton number ∴ not possible for $e^{\pm}$, etc

which is gauge invariant. However, it has mass dimension 5, so the coefficient in front is proportional to $1/M_{Planck}$. Thus, we expect that m_ν in the SM is (at most) of order

$$\langle 100\ \text{GeV}\rangle^2 / M_{Planck} \sim 10^{-5}\ \text{eV}$$

(or less).

Neutrino Oscillations

For non-vanishing neutrino masses, and if lepton number is not absolutely conserved, the weak-interaction eigenstates are linear combinations of mass eigenstates.

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

produced in weak intcns. (sun, atmosphere, reactors)

Due to their mass differences, ν_1 and ν_2 propagate with slightly different frequencies:

$$\nu_1(t) = \nu_1(0)\, e^{-iE_1 t}$$

$$\nu_2(t) = \nu_2(0)\, e^{-iE_2 t}$$

eV^2 meters

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \, \sin^2\left(\frac{1.27\, \Delta m^2 L}{E}\right)$$

MeV

For solar ν's care must be exercised as they traverse the sun's interior (MSW).

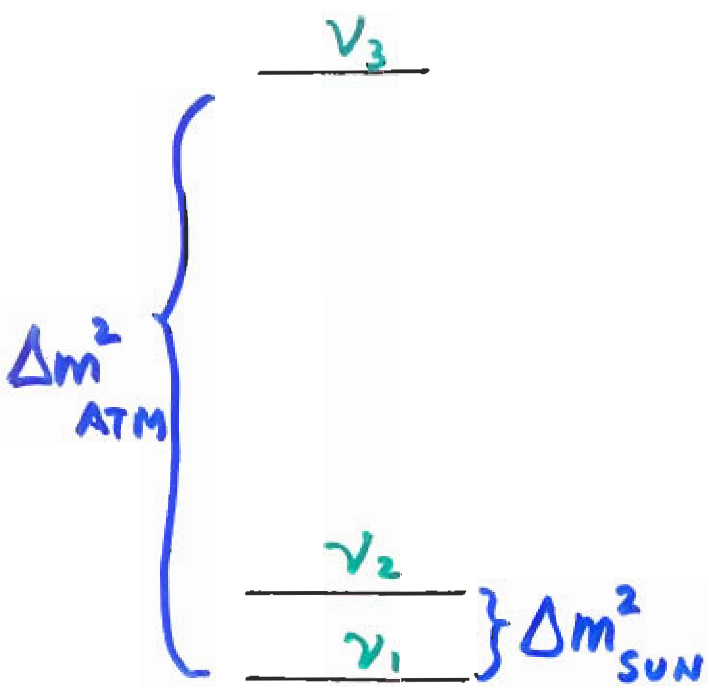
$$\Delta m^2_{ATM} \simeq 2.5 \times 10^{-3}\, eV^2$$

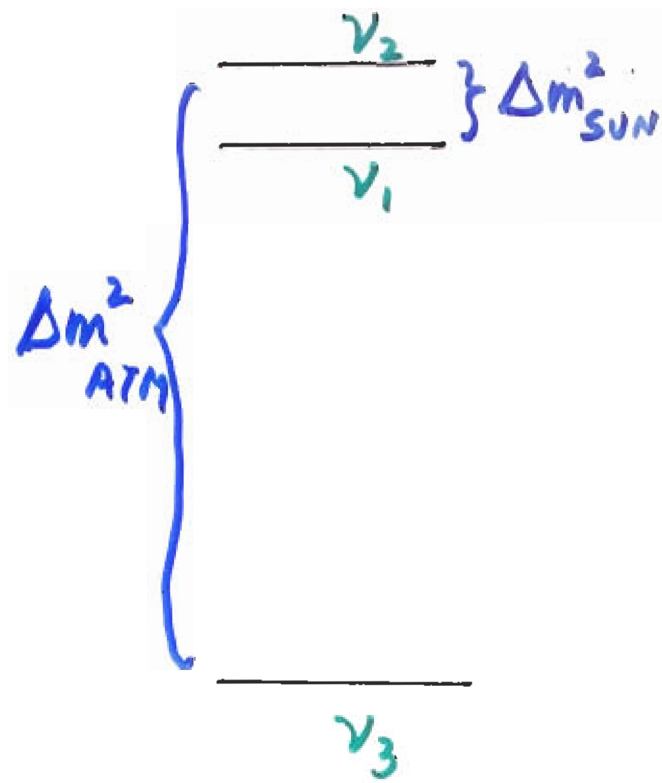
$$\Delta m^2_{SOL} \simeq 8 \times 10^{-5}\, eV^2$$

$$\left\{ \begin{array}{l} Cf: \\ SM \end{array} \right. \frac{1}{M_P} \underbrace{(L H)^2}_{\uparrow\ \text{dim5 in SM}} \Rightarrow m_\nu \lesssim 10^{-5} eV$$

$\Rightarrow$ new physics between

M_W and M_P.





(Neutrinos could also exhibit mass degeneracy)

What about the absolute values of ν masses?

- Tritium β decay expt:

$$m_\beta = \left[\sum |U_{ei}|^2 m_i^2\right]^{1/2} \leq 2.2\,\text{eV}$$

- WMAP + Large Scale Structure

$$\sum m_{\nu_i} \lesssim 1\,\text{eV}$$

See-Saw Mechanism

Neutrinos essentially 'massless' in SM because ν_R is absent. By introducing ν_R, we could have a Dirac mass term $h\,\bar{\nu}_R\,\nu_L\,H \sim 10^{-1}\,eV$, provided $h \sim 10^{-12}$. This is not particularly attractive (unless we invoke an extra dimension).

Note that ν_R is a SM singlet

$\Rightarrow$ we can have a mass term

violates L $\longrightarrow$ $M\,\nu_R\,\nu_R$ $(M \gg M_W)$

The neutrino mass matrix

$$\begin{array}{c|cc} & \nu_L & \nu_R \\ \hline \nu_L & & \\ \nu_R & & \end{array}
\quad
\begin{array}{cc} & \\ \nu_L & \\ \nu_R & \end{array}
\begin{array}{cc} \nu_L & \nu_R \\ \left(\begin{matrix} \approx 0 \\ m_{Dirac} \end{matrix}\right. & \left.\begin{matrix} m_{Dirac} \\ M \end{matrix}\right) \end{array}$$

Eigenvalues:

$$\lambda_{LARGE} \approx M$$

$$\lambda_{SMALL} \approx m_{Dirac} \frac{m_{Dirac}}{M}$$

$$\sim 10^{-1}\,\mathrm{eV} \quad \text{if } M \sim 10^{14}\,\mathrm{GeV}$$

NEW PHYSICS

Neutrinos are Majorana particles

$\Rightarrow \beta\beta 0\nu$

Neutrino Sector

- $\Delta m^2_{ATM} \simeq 2.5 \times 10^{-3}\, eV^2$

  $\sin^2 2\theta_{23} \simeq 1$

- $\Delta m^2_{SOL} \simeq 8 \times 10^{-5}\, eV^2$

  $\sin^2 2\theta_{12} \simeq 0.8$

- $\theta_{13} \lesssim 0.2$ (CHOOZ)

Charged Leptons exhibit mass hierarchies: $\lambda_e : \lambda_\mu : \lambda_\tau \sim \lambda^5 : \lambda^2 : 1$,

with $\lambda \simeq 0.2$

Yukawa Couplings

Also, quark masses and mixings (CKM) exhibit a hierarchical structure:

$$\lambda_u : \lambda_c : \lambda_t \sim \lambda^8 : \lambda^4 : 1$$

$$\lambda_d : \lambda_s : \lambda_b \sim \lambda^4 : \lambda^2 : 1$$

$$V_{us} \simeq \lambda \,, \; V_{cb} \sim \lambda^2, \; V_{ub} \sim \lambda^3 - \lambda^4 .$$

- Origin of this ?

- CKM mixings small, while two ν mixings are large; how ?

- Usual approach relies on postulating an additional flavor symmetry G_F which can be Abelian, non-Abelian (discrete &/or continuos).

- Simplest example based on $U(1)$

  Froggat, Nielsen

  $U(1)$ breaking yields $\frac{\langle X \rangle}{M_*} \equiv \lambda \simeq 0.2$,

  and suitable choice of $U(1)$ flavor charges should yield the observed masses & mixings.

- In practice, though, things are not so simple.

Example : $SU(5) \times U(1)$

Fermions : $10\,(q, u^c, e^c)$, $\bar{5}\,(d^c, \ell)$, $1\,(\nu^c)$

CKM & down quark / charged lepton mass hierarchies more or less fix $U(1)$ charges :

$$Q(10_\alpha) = (3, 2, 0), \quad Q(\bar{5}_\alpha) = (1, 0, 0)$$

$$\begin{array}{cc} & \begin{array}{ccc} \bar{5}_1 & \bar{5}_2 & \bar{5}_3 \end{array} \\ \begin{array}{c} 10_1 \\ 10_2 \\ 10_3 \end{array} & \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} \bar{H}(\bar{5}) \end{array}$$

$V_{us} \sim \lambda$, $V_{cb} \sim \lambda^2$, $V_{ub} \sim \lambda^3$ (decent)

$10_2 - 10_3$ mixing is small

$\Updownarrow$

$\bar{5}_2 - \bar{5}_3$ mixing is large

$\Rightarrow$ large $\ell_2 - \ell_3$ mixing (good)

However, $\ell_1 - \ell_2$ mixing from this sector is small $(\sim \lambda)$.

Also, at M_{GUT}, $M_D = M_E$

$\Rightarrow \quad m_b = m_\tau$ (okay)

$m_d / m_s = m_e / m_\mu$ (bad!)

$U(1)$ Scenarios

Frogatt, Nielsen
⋮

- (Quasi) degenerate neutrinos

$$m_1 \simeq m_2 \simeq m_3$$

Difficult with $U(1)$; $SO(3)/SU(3)$ more promising; RG instabilities.

- ~~Inverted~~ Normal Hierarchy

$$m_1 << m_2 << m_3 \sim \sqrt{\Delta m^2_{ATM}}\,;\; m_2 \sim \sqrt{\Delta m^2_{SOL}}$$

Stable under RG;

- Inverted Hierarchy

$$m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{ATM}} \;;\; m_3 \approx 0$$

Stable under RG; 'maximal' θ_{12} requires tuning.

Democratic Approach

- $U(1)$ does not distinguish ℓ_1, ℓ_2, ℓ_3

  $\Rightarrow$ large lepton mixings (Fukugita, Tanimoto et. al., Murayama, ...)

  θ_{12} too large ?

  θ_{13} ?

  m_{ν_i} ?

- Single RHN dominance (Suematsu, King, ...)

  can yield $m_{sol}/m_{atm} \ll 1$ (!)

- (with double seesaw)

  In a more elaborate scheme, applicable in some GUTs,

$$\tan\theta_{13} \simeq \sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} \; \frac{\tan\theta_{12}\tan\theta_{23}}{1+\tan^2\theta_{12}}$$

$$\approx 0.05 - 0.14$$

(Q.S + Z. Tavartkiladze)

μ - τ Symmetry

(Near) maximal atmospheric neutrino mixing has motivated the mass matrix (μ-τ symmetry):

$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}, \quad x, y, z \in \mathbb{C}$$

From this one finds

$$\theta_{23} = 45^\circ$$

$$\theta_{13} = 0^\circ$$

- No prediction for neutrino masses; include quarks?

Matrix V which diagonalizes the neutrino mass matrix is proportional to

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & 1/\sqrt{2} \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & -1/\sqrt{2} \end{pmatrix}$$

For $\sin^2\theta = 1/3$ (solar ν osc), one obtains the tri-bimaximal mixing matrix

$$V \propto \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Models 'based' on A_4 (even permutations of 4 objects) attempt to realize this mixing pattern.

Ingredients include several new scalars as well as additional symmetries such as $U(1)$, Z_n and $U(1)_R$ (in susy case).

A_4 Example

Ma et. al.
(Altarelli et. al)
⋮

$$\mathcal{L}_Y = y_e e^c (\phi \ell) h_d/\Lambda + y_\mu \mu^c (\phi\ell)'' h_d/\Lambda$$

$$+ y_\tau \tau^c (\phi \ell)' h_d/\Lambda + x_a \xi (\ell h_u)^2/\Lambda^2$$

$$+ x_d (\phi' (\ell h_u)^2) + \text{h.c.} + \text{higher order}$$

(Λ = suitable cutoff)

Add'l symmetries imposed to banish undesirable terms such as $(\ell h_u)^2$, $\phi' \leftrightarrow \phi$ exchange, etc.

'Require' $\langle \phi' \rangle = (v', 0, 0)$, $\langle \phi \rangle = (v, v, v)$,
$\langle \xi \rangle = u$ (ϕ, ϕ' are A_4 triplets; ξ = A_4 singlet)

This yields

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

$$m_\ell = \frac{v_d v}{\Lambda}\begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu \omega^2 & y_\mu \omega \\ y_\tau & y_\tau \omega & y_\tau \omega^2 \end{pmatrix}$$

with $a \equiv x_a \frac{u}{\Lambda}$, $d \equiv x_d \frac{v'}{\Lambda}$; $\omega = e^{2\pi i/3}$

Introducing $U(1)_F$ (charges $0, 2, 3 \div 4$ for τ^c, μ^c, e^c)

and completing calculation yields

$$\tan^2\theta_{23} = 1$$

$$\tan^2\theta_{12} = 0.5$$

$$\theta_{13} = 0$$

Also, with moderate fine tuning, one sol^n yields

$|m_3| \approx 0.05\ \mathrm{eV}$, $|m_1| \simeq |m_2| \approx 0.017\ \mathrm{eV}$

BARYONS FROM LEPTONS

According to Sakharov to realize baryon asymmetry one should have:

- B violation
- Departure from thermal equilibrium
- C and CP violation

The SM has these ingredients but things dont quite work out:

$m_{higgs} \lesssim 70$ GeV (Expt: $m_h \gtrsim 114$ GeV)

CP violation suppressed

Require physics beyond the SM.

In one popular scenario (Fukugita & Yanagida), baryons come from leptons.

- An initial lepton asymmetry arises from the decay of RH neutrinos:

$$N \to L H \,, \quad N \to \bar{L} \bar{H}$$

(and interference between tree & loop diagrams)

- EW sphalerons are exploited to achieve the desired baryon asymmetry

(An initial lepton asymmetry is partially converted by sphalerons into baryon asymmetry]

- Non-thermal leptogenesis can be easily realized through inflation models.

In one simple scenario the inflaton decay yields RH neutrinos $\Longrightarrow$ leptogenesis.

- Example: Spontaneous breaking of (gauged) $U(1)_{B-L}$

  $\downarrow$ requires RH $\nu_R (N)$

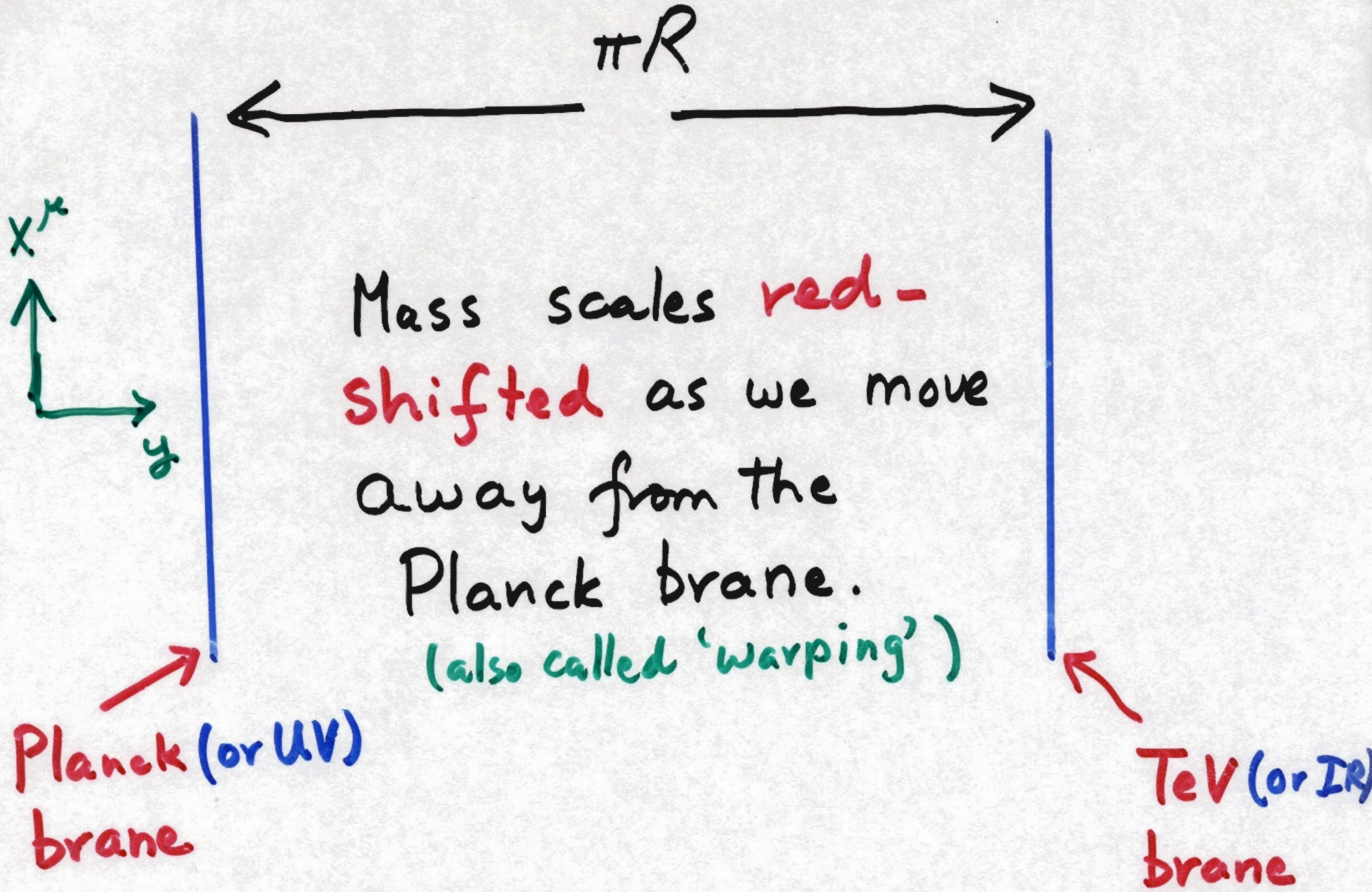
susy models work especially well here.

WARPED EXTRA DIMENSION(S)

Introduce a bulk cosmological constant Λ in 5D such that

$$ds^2 = e^{-2k|y|} \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{\text{3+1 dim Minkowski space}} + \underset{\substack{\uparrow \\ 5^{\text{th}} \\ \text{coordinate}}}{dy^2}$$

$\Lambda < 0$ (anti deSitter space)



$(\text{TeV} \simeq M_p e^{-\pi k R})$

$(kR \sim 10)$

TWO SCENARIOS

- Gravity propagates in bulk; SM fields reside on TeV brane.
  (especially Higgs)
- 'All' fields allowed to propagate in bulk $\Rightarrow$ many interesting consequences.

except Higgs!

Original Proposal

All SM fields reside on the TeV brane ; only gravity feels the extra dimension.

$\Rightarrow$ hierarchy problem under control!
(scale on TeV-brane ~ TeV)

BUT

Difficulty with non-renormalizabe operators :

$$\frac{1}{M_{Pl}^2}\,\bar{\Psi}\Psi\bar{\Psi}\Psi \longrightarrow \frac{1}{(\mathrm{TeV})^2}\,\bar{\Psi}\Psi\bar{\Psi}\Psi$$

→ Rapid p decay

→ Large FCNC

→ Large neutrino masses ($\gg$ eV)

→ Fermion Mass hierarchies & mixings ?

WAY OUT ?

- Symmetries ?

- Permit SM fields to leave the brane ? (Higgs stays on the TeV brane)

unless SUSY invoked

Gherghetta
Pomarol
Hewett et al
Huber + QS
⋮

5d Bulk field ⟹ Tower of 4d fields

KK ansatz: $\Phi(x^\mu, y) = \sum_{n=0}^{\infty} \Phi^n(x^\mu) f_n(y)$

wave fns from field eq^{ns}

Neutrino Oscillations in RS

(Dirac) ν

- 3 right-handed ν's in bulk
- Dirac mass from

Grossman, Neubert, ⋮

$$\int d^4x \int dy \sqrt{-G} \; \frac{\lambda^{(5)}_{ij}}{\sqrt{k}} \; H \; \bar{L}^i \; \psi^j$$

TeV-brane

'Bulk'

RH-ν near Planck brane

!! (Dirac masses suppressed)

Spectrum (flavor index suppressed)

$$\text{SM } \nu\text{'s : KK-tower} \begin{cases} \nu_L^{(0)}, & \nu_L^{(1)}, \; \nu_L^{(2)}, \ldots \\ -, & \nu_R^{(1)}, \; \nu_R^{(2)}, \ldots \end{cases}$$

$$\text{Sterile } \nu\text{'s (Right-handed)} \begin{cases} -, & \psi_L^{(1)}, \; \psi_L^{(2)}, \ldots \\ \psi_R^{(0)}, & \psi_R^{(1)}, \; \psi_R^{(2)}, \ldots \end{cases}$$

Neutrino Mass Matrix

$$
\begin{array}{c|cccc}
 & \psi_R^{(0)} & \nu_R^{(1)} & \psi_R^{(1)} & \cdots \\
\hline
\nu_L^{(0)} & \langle H\rangle & 0 & \langle H\rangle & \cdots \\
\nu_L^{(1)} & \langle H\rangle & M & \langle H\rangle & \cdots \\
\psi_L^{(1)} & 0 & \langle H\rangle & \underset{\uparrow\ \text{KK scale}}{M} & \\
\vdots & \vdots & \vdots & & \ddots
\end{array}
$$

For solar ν both small and large mixing angle solutions can be realized, depending on how we choose the 'c' parameters.

Consider bimaximal mixing

ν_L: $c_{e_L} = c_{\mu_L} = c_{\tau_L} = 0.57$ } ← Explains $m_\tau \gg m_\mu \gg m_e$

$(c_{e_R} = 0.79,\ c_{\mu_R} = 0.62,\ c_{\tau_R} = 0.50)$

ψ_R: $c_{\psi_1} = 1.43,\ c_{\psi_2} = 1.36,\ c_{\psi_3} = 1.30$

Large MSW

$$\lambda_{ij} = \begin{pmatrix} -2.0 & 1.5 & -0.5 \\ -1.8 & -1.1 & 1.9 \\ 0.5 & 1.9 & 1.7 \end{pmatrix}$$

5-d coupling

$\Delta m^2_{atm} = 5 \cdot 10^{-3}\ eV^2 \rightarrow$ atm $(\sim m_{\nu_1})$

$\Delta m^2_{sol} = 1.\ 10^{-4}\ eV^2 \rightarrow$ Solar $(\sim m_{\nu_2})$

$\sin^2 2\theta_{atm}$ $0.98 \rightarrow$ atm

$\sin^2 2\theta_{sol} \simeq 0.90 \rightarrow$ Solar

$[U^2_{e3} = 0.036]$ close to exp bound

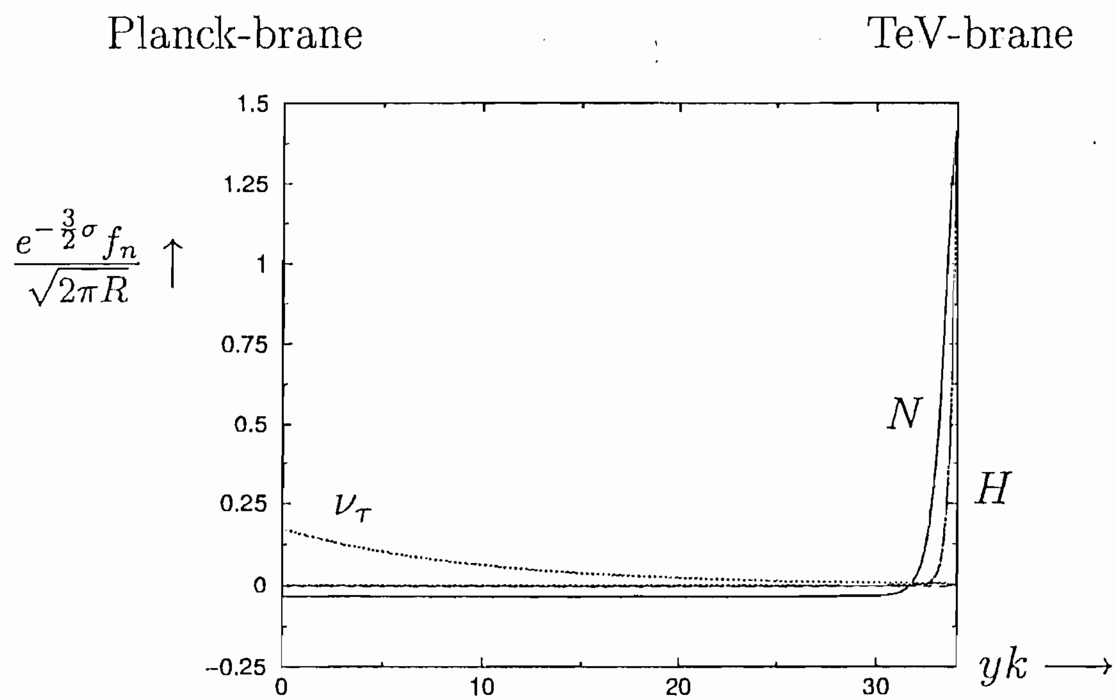
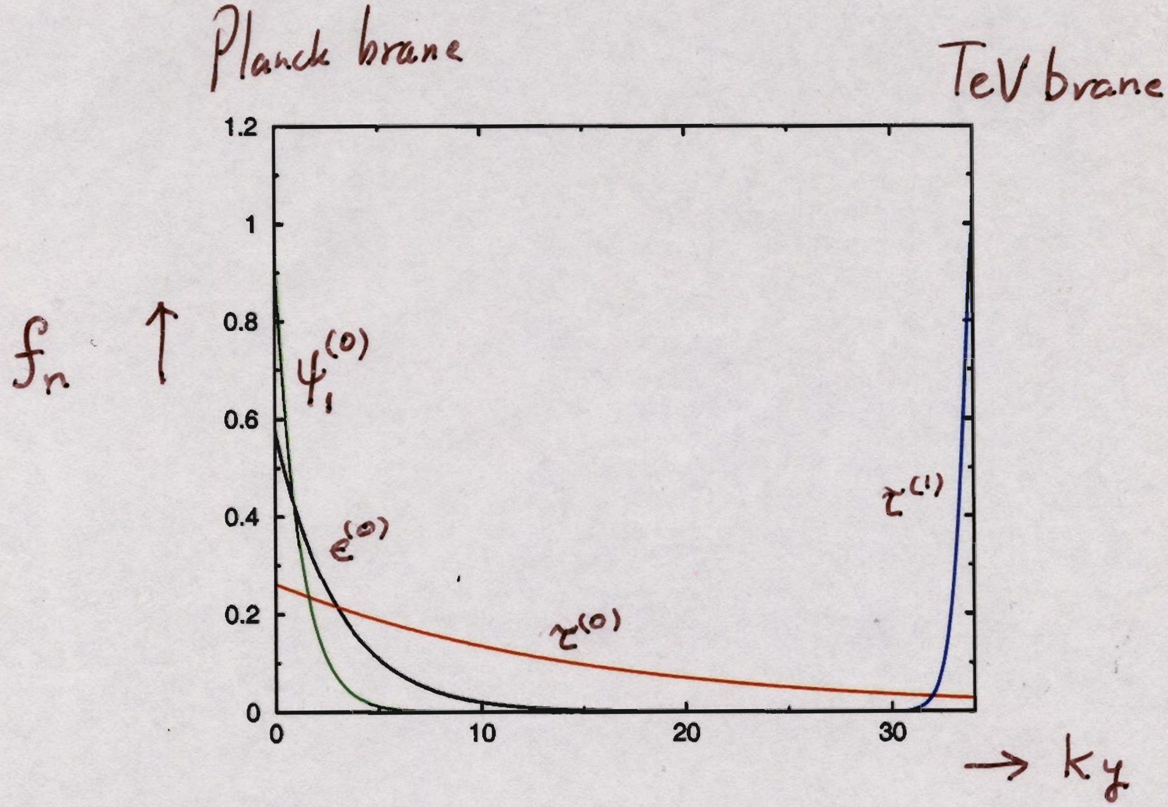


Figure 1: Wave functions of the ν_τ and N zero modes, together with a sketchof the Higgs profile.



Planck brane
TeV brane
f_n ↑
$\psi_1^{(0)}$
$e^{(0)}$
$\tau^{(0)}$
$\tau^{(1)}$
→ ky
1.2
1
0.8
0.6
0.4
0.2
0
10
20
30

Majorana Case Consider the dimension five operator

$$\int d^4x \int dy \sqrt{-g}\ \frac{l_{ij}}{M_5^2}\ H^2\ \Psi_i\ C\ \Psi_j$$

Charge conjugation

$$\equiv \int d^4x\ M_{ij}^{(\nu)}\ \Psi_i^{(0)}\ C\ \Psi_j^{(0)},$$

$$M_{ij}^{(\nu)} = \int_{-\pi R}^{+\pi R} \frac{dy}{2\pi R}\ \frac{l_{ij}}{M_5^2}\ e^{-4\sigma(y)}\ H^2(y)\ f_{0i}^{(\nu)}(y)\, f_{0j}^{(\nu)}(y).$$

With SM fields in the bulk, dim 5 ν masses can become unacceptably large if they stray too close to the TeV brane.

A careful choice of k/M_5 (~ 0.01) and l ($\sim 10^{-3}$) can yield neutrino masses

See Saw Mechanism in Warped Geometry

In ordinary seesaw

$$m_\nu \sim m_D^2 / M_R$$

Thus, $M_R \sim$ intermediate scale to generate m_ν's relevant for ν oscillations;

Intermediate scale M_R is also appropriate for implementing Leptogenesis (thermal or non-thermal).

? How to generate such intermediate scales in a warped background.

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{-G} \left\{ \overline{\Psi} i E_a^{\ M} \gamma^a (\partial_M + \omega_M) \Psi \right.$$

$$\left. - m_D \overline{\Psi} \Psi - m_M \overline{\Psi} \Psi^c \right\}$$

curved space index

$E_a^{\ M} \rightarrow$ fünfbein

$\gamma^a \rightarrow (\gamma^\mu, \gamma^5)$ ← Dirac matrices in flat space

$\omega_M = \left(\frac{1}{2} \sigma' e^{-\sigma} \gamma^5 \gamma_\mu, 0 \right), \sigma' = d\sigma/dy$

$\Psi^c = C_5 \gamma^0 \Psi^*$

$m_D = -c\sigma'$ (Dirac mass)

$m_M =$ Majorana mass

Consider two bulk fermion fieds ν and N corresponding to left and right handed neutrinos.

Their bulk Dirac masses are

Charcterized by c_ν and c_N.

- If a Majorana mass ~~for~~ N of order M_p is localized on the TeV brane, it gets warped down to TeV size; expect to find 'light' masses

$$\sim \mathrm{GeV}^2/\mathrm{TeV} \sim \mathrm{MeV}$$

- With a Majorana mass for N (of order M_P) localized on the Planck brane, one can generate intermediate scale masses of order

$$M_P \exp\left\{(2c_N - 1)\pi k R\right\}, \quad (\underset{\substack{\uparrow \\ \text{gauge} \\ \text{hierarchy}}}{kR \simeq 11})$$

For $c_N \simeq 0.3$, the intermediate mass is of order 10^{12} GeV. Thus, ~~if~~ it appears that seesaw mechanism can be realized in a warped background. Models? Leptogenesis?

SUMMARY (ν Oscillations & Warped Geometry)

- Bulk SM fields;
- Higgs on the TeV Brane;
- Neutrinos could be

Dirac or Majorana; $\longrightarrow$ seesaw

$\downarrow$ introduce SM singlet fermion (eliminate dim 5 Majorana masses by imposing some symmetry, say lepton number) $\Rightarrow$ p stable;

But $n-\bar{n}$ possible;

Also $\mu \to e\gamma$, etc.

$\downarrow$ dim 5 operators (no new particles introduced)

$\downarrow$ $\beta\beta 0\nu$ possible;

Lepton no. parity would allow dim 5 ops. but suppress p decay.

- SMOKING GUN:

FIND K-K excitations of SM gauge fields (at LHC?)

(• GRAND UNIFICATION?)

SM + Einstein's GR leads to highly successful hot big bang cosmology :

- Blackbody nature of cosmic microwave background ;
  $T = 2.725\ K$ (also $C\nu B$).

- Redshift of galaxies ;

- Primordial abundance of light elements .

Universe undergoes phase transitions:

- EW at $T \sim 10^{2}\ GeV$, $t \sim 10^{-10}\ sec$
- QCD at $T \sim 10^{-1}\ GeV$, $t \sim 10^{-4}\ sec$

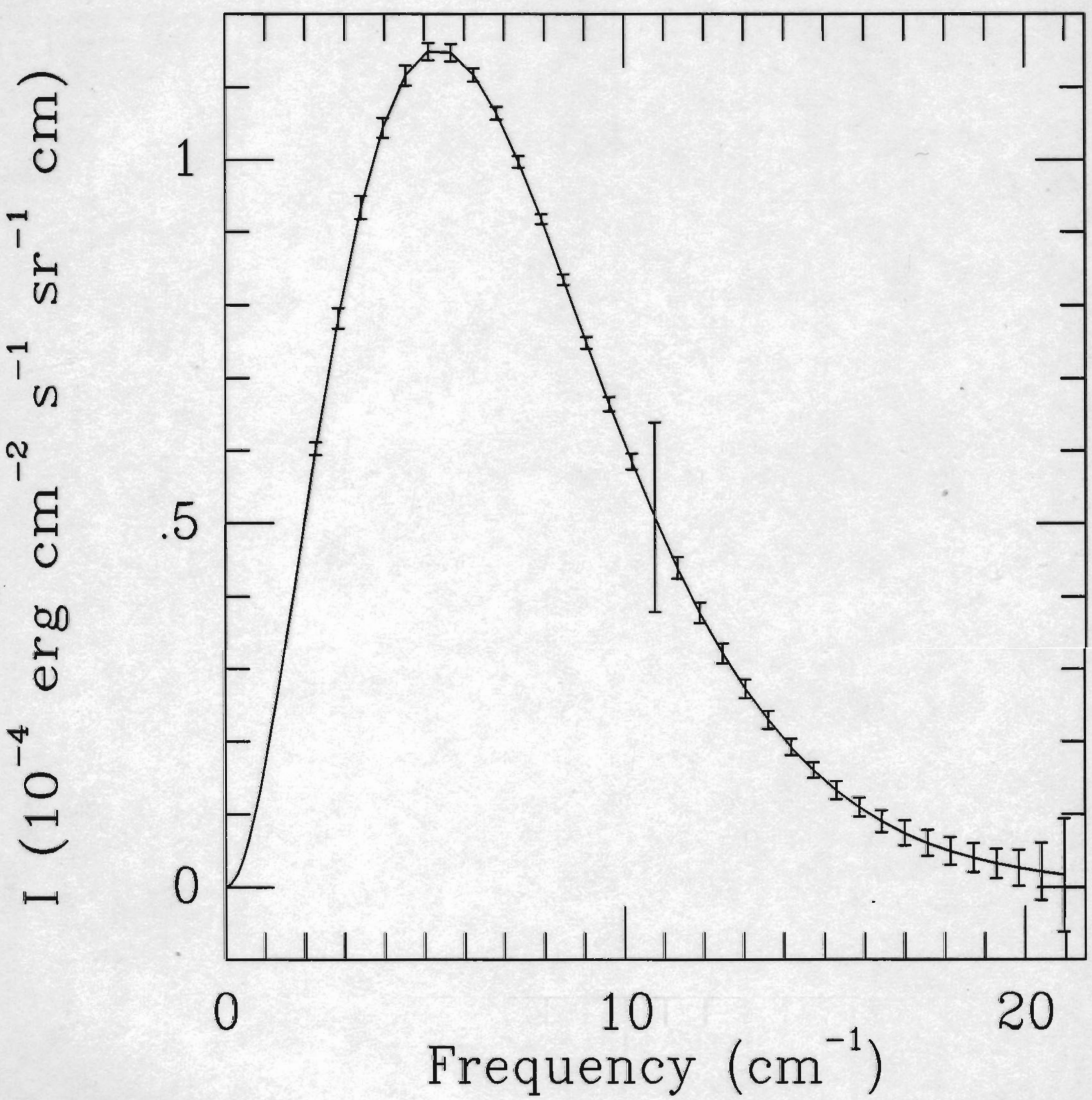


Figure 4: Spectrum of the Cosmic Microwave Background Radiation as measured by the FIRAS instrument on COBE and a black body curve for $T = 2.7277\,\mathrm{K}$. Note, the error flags have been enlarged by a factor of 400. Any distortions from the Planck curve are less than 0.005% (see Fixsen *et al.* , 1996).

WHY INFLATION?

i) **Horizon Problem**

The cosmic background radiation is highly istropic. How did this come about?

Measurement of CBR gives us a snapshot of the universe at $T \sim 5 \times 10^3$ K. If we observe the CBR with antennae pointing in opposite dir^{ns} in the sky you are looking at regions separated by 90 (or so) 'horizon' lengths.

PUZZLE: Why do we still measure $T_0 \simeq 2.728^\circ$K ??

ii) The observable universe contains about 10^{90} particles.

Where did they come from?

iii) The microwave radiation shows some degree of anisotropy,

$$\Delta T/T\Big|_{(\theta \sim 10^\circ \to 90^\circ)} \sim \text{few} \times 10^{-6}$$

What is the origin of this?

Are they somehow related to the rest of the observed structure?

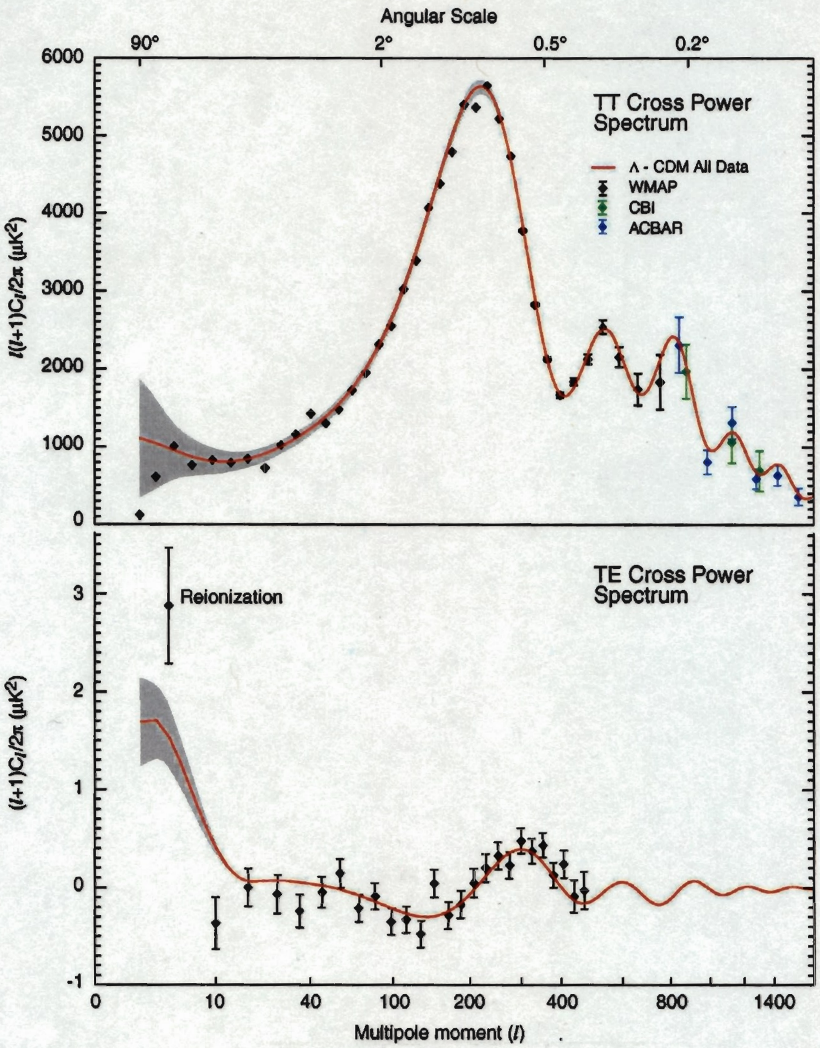


Fig. 12.— The *WMAP* angular power spectrum. (*top*:) The *WMAP* temperature (TT) results are consistent with the ACBAR and CBI measurements, as shown. The TT angular power spectrum is now highly constrained. Our best fit running index ΛCDM model is shown. The grey band represents the cosmic variance expected for that model. The quadrupole has a surprisingly low amplitude. Also, there are excursions from a smooth spectrum (e.g., at $\ell \approx 40$ and $\ell \approx 210$) that are only slightly larger than expected statistically. While intriguing, they may result from a combination of cosmic variance, subdominant astrophysical processes, and small effects from approximations made for this first year data analysis (Hinshaw et al. 2003b). We do not attach cosmological significance to them at present. More integration time and more detailed analyses are needed. (*bottom*:) The temperature-polarization (TE) cross-power spectrum, $(l+1)C_l/2\pi$. (Note that this is *not* multiplied by the additional factor of l.) The peak in the TE spectrum near $l \sim 300$ is out of phase with the TT power spectrum, as predicted for adiabatic initial conditions. The antipeak in the TE spectrum near $l \sim 150$ is evidence for superhorizon modes at decoupling, as predicted by inflationary models.

$$\Delta T/T\,(\theta,\phi) = \sum a_{\ell m} Y_{\ell m}(\theta,\phi)$$

$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle$$

One resolution of these puzzles is obtained by invoking INFLATION (Guth)

Let us suppose that the very early universe had an epoch in which the scale factor $R(t)$ underwent acceleration. From

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p)$$

ρ: energy density

p: pressure

$\Rightarrow \quad p < -\rho/3 \quad (\text{for } \ddot{R} > 0)$

$\Rightarrow$ inflation (acceleration) wont take place in a radiation or matter dominated universe.

Suppose, however, that the very early universe is dominated by a cosmological constant.

This could arise from a homogeneous scalar field S such that

$$\rho_S = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_S = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

If the potential term dominates,

$$p_S \approx -\rho_S$$

and

$$R(t) \propto e^{Ht} \quad , \quad H^2 = \frac{8\pi}{3} G \rho_S$$

$\Rightarrow$ inflationary epoch

Quartic (CW) Potential (non-susy)

$$V(\phi) = A\phi^4\left[\ln\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + \frac{AM^4}{4}$$

gauge singlet

$$V(\phi = M) = 0\ ; \quad V(\phi = 0) = AM^4/4 \equiv V_0$$

$$V(\phi \ll M) \simeq \frac{AM^4}{4} - b\phi^4$$

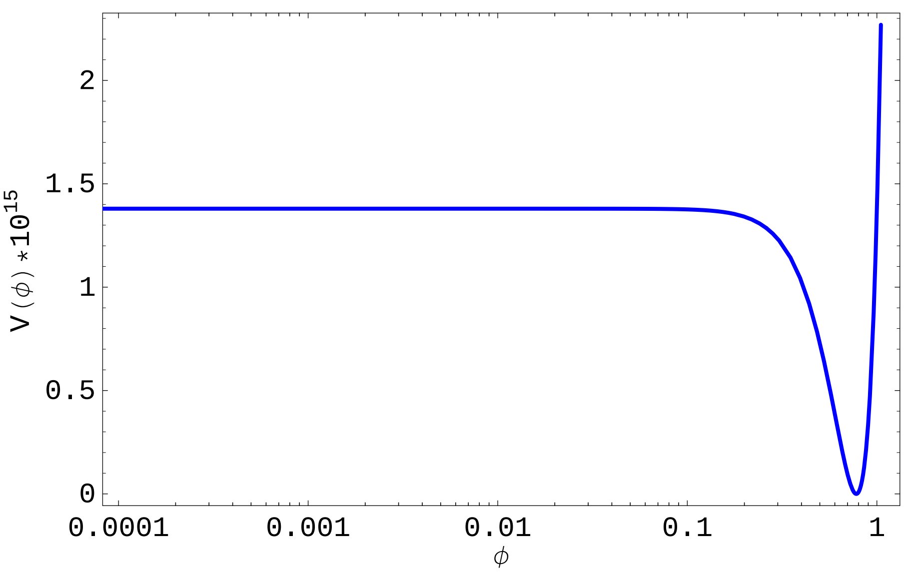
- For $V_0^{1/4} < 10^{16}$ GeV, $\phi < m_p$ ($\simeq 2.4 \times 10^{18}$ GeV)

Model behaves as for $V \simeq V_0(1 - (\phi/\mu)^4)$

$$n_s \simeq 1 - \frac{3}{N_0}, \quad \alpha \simeq (n_s - 1)/N_0$$

e-folds for $k_0 = 0.002\ \mathrm{Mpc}^{-1}$

($V_0^{1/4} > 10^5$ GeV to avoid conflict with WMAP)



The Coleman-Weinberg potential $V(\phi) = A\phi^4 \left[\ln\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + \frac{AM^4}{4}$ for $V_0^{1/4} = 5 \times 10^{14}$ GeV ($m_P = 1$ in the plot).

WMAP 3 : $n_s \approx 0.95 \pm 0.02$

Other : $n_s \approx 0.965 \pm 0.02$ (?)

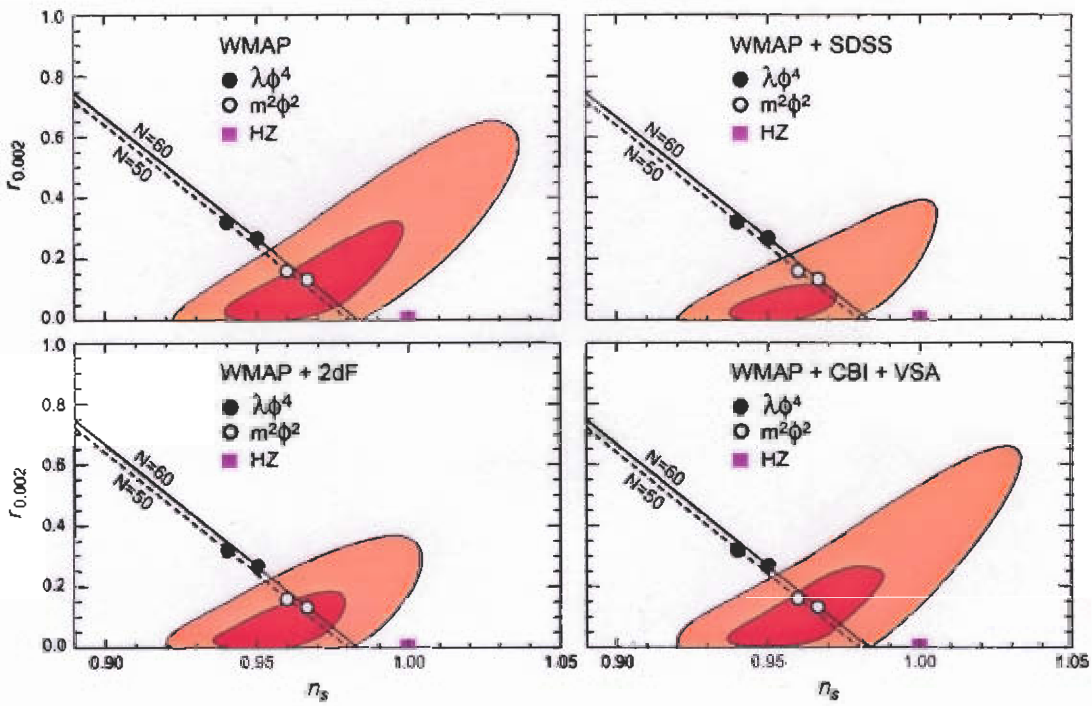


Fig. 14.— Joint two-dimensional marginalized contours (68% and 95% confidence levels) for inflationary parameters ($r_{0.002}$, n_s) predicted by monomial potential models, $V(\phi) \propto \phi^n$. We assume a power-law primordial power spectrum, $dn_s/d\ln k = 0$, as these models predict the negligible amount of running index, $dn_s/d\ln k \approx -10^{-3}$. (*Upper left*) WMAP only. (*Upper right*) WMAP+SDSS. (*Lower left*) WMAP+2dFGRS. (*Lower right*) WMAP+CBI+VSA. The dashed and solid lines show the range of values predicted for monomial inflaton models with 50 and 60 e-folds of inflation (equation (13), respectively. The open and filled circles show the predictions of $m^2\phi^2$ and $\lambda\phi^4$ models for 50 and 60 e-folds of inflation. The rectangle denotes the scale-invariant Harrison-Zel'dovich-Peebles (HZ) spectrum ($n_s = 1, r = 0$). Note that the current data prefers the $m^2\phi^2$ model over both the HZ spectrum and the $\lambda\phi^4$ model by likelihood ratios greater than 50.

(WMAP 1 : $n_s \approx 0.99 \pm 0.04$)

Other Analyses : $n_s \approx 0.98 \pm 0.02$

- For $V_0^{1/4} \gtrsim 10^{16}$ GeV, $\phi > m_p$ during observable inflation.

Predictions approach that of ϕ^2 potential, with

$$n_s = 1 - \frac{2}{N_0} \simeq 0.96$$

$$r \simeq 0.13$$

$$\alpha \simeq -0.6 \times 10^{-3}$$

? Where does ϕ come from

Breaks global $U(1)_{B-L}$ (SM)

$U(1)_{PQ}$

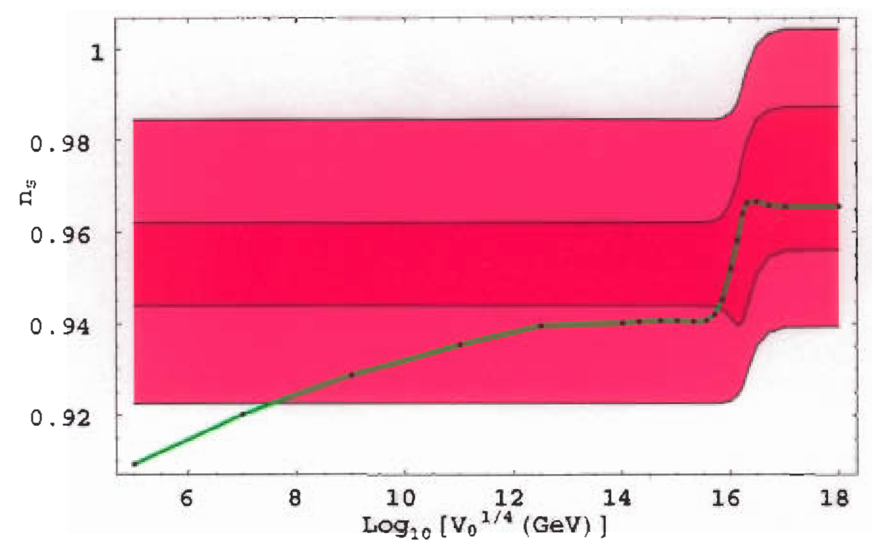


FIG. 1: The spectral index n_s vs $\log[V_0^{1/4}\,(\mathrm{GeV})]$ for the Coleman-Weinberg potential (green curve), compared with the WMAP range for n_s (68% and 95% confidence levels, taken from Spergel *et al.*, astro-ph/0603449). Note that the tensor to scalar ratio $r \approx 0$ for $V_0^{1/4} \ll 10^{16}$ GeV and $r \approx 0.14$ for $V_0^{1/4} \gg 10^{16}$ GeV.

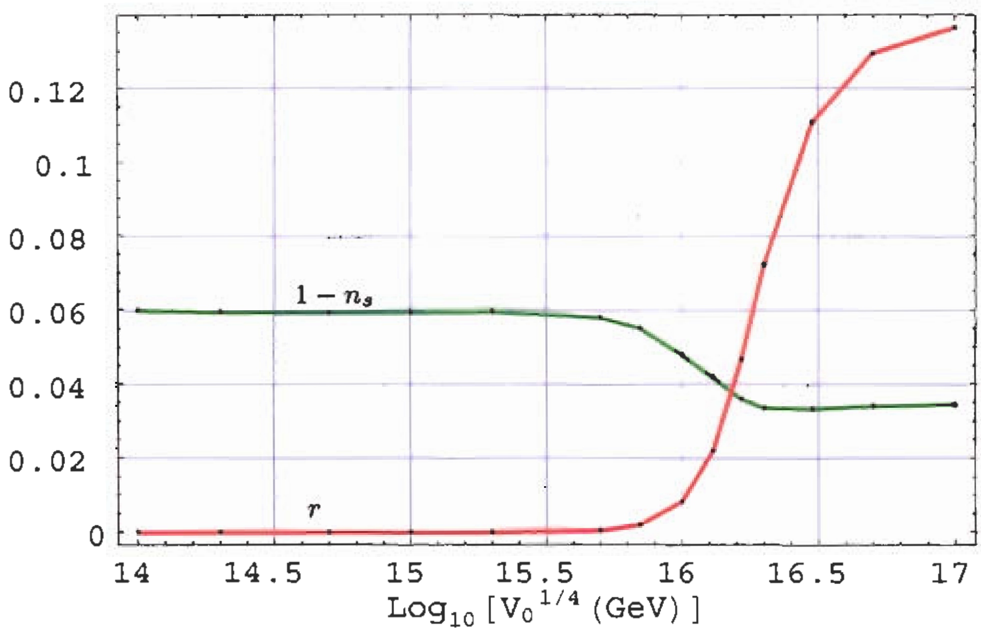


FIG. 1: $1 - n_s$ and r vs. $\log[V_0^{1/4}\,(\mathrm{GeV})]$ for Coleman-Weinberg potential.

? Lower Bound on V_0

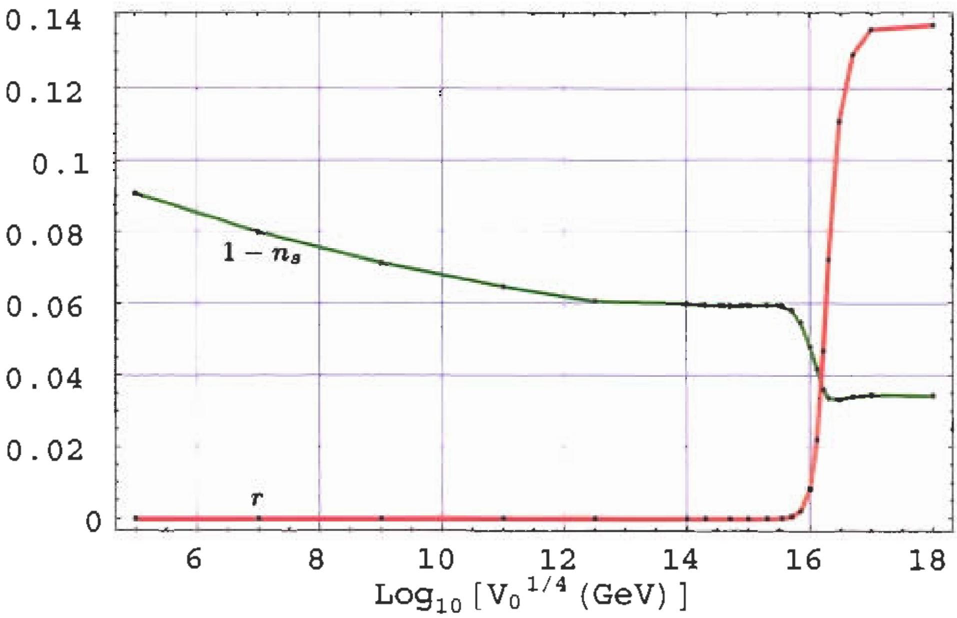


FIG. 1: $1 - n_s$ and r vs. $\log[V_0^{1/4}\,(\mathrm{GeV})]$ for the Coleman-Weinberg potential.

Table 1: The inflationary parameters for the Shafi-Vilenkin model ($m_P = 1$)

$V_0^{1/4}$(GeV)	$A(10^{-14})$	M	ϕ_e	ϕ_0	$V(\phi_0)^{1/4}$(GeV)	n_s	α $(-)10^{-3}$	r
10^{13}	1.0	0.018	0.010	3.0×10^{-6}	$\approx V_0^{1/4}$	0.938	1.4	9×10^{-15}
5×10^{13}	1.2	0.088	0.050	7.5×10^{-5}	$\approx V_0^{1/4}$	0.940	1.3	5×10^{-12}
10^{14}	1.3	0.17	0.10	3.0×10^{-4}	$\approx V_0^{1/4}$	0.940	1.2	9×10^{-11}
5×10^{14}	1.9	0.79	0.51	7.5×10^{-3}	$\approx V_0^{1/4}$	0.941	1.2	5×10^{-8}
10^{15}	2.3	1.5	1.1	0.030	$\approx V_0^{1/4}$	0.941	1.2	9×10^{-7}
5×10^{15}	4.8	6.2	5.1	0.71	$\approx V_0^{1/4}$	0.942	1.0	5×10^{-4}
1×10^{16}	5.2	12	10	3.2	9.9×10^{15}	0.952	1.0	8×10^{-3}
2×10^{16}	1.1	36	35	23	1.7×10^{16}	0.966	0.6	0.07
3×10^{16}	.17	86	85	72	1.9×10^{16}	0.967	0.6	0.11

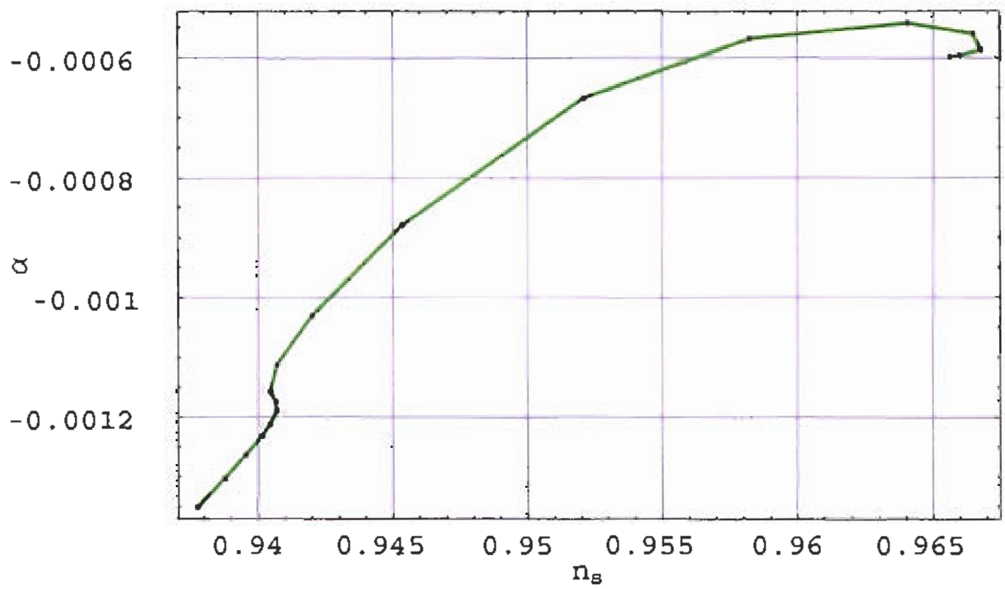


FIG. 2: α vs. n_s for the Coleman-Weinberg potential.

Let ϕ denote the inflaton field with vev which breaks $U(1)_{B-L}$. Decay of ϕ occurs via

$$h_{ij}\,\phi\, N_i N_j\,, \qquad h_{ij} \sim \frac{m_N}{\langle\phi\rangle}$$

RH neutrino

The observed baryon asymmetry arises via leptogenesis.

Conclusions

- Quartic (CW) potential yields predictions in good agreement with WMAP3.
  - When applied to SO(10) broken, say, via $SU(4) \times SU(2) \times SU(2)$, one finds that doubly charged monopoles may exist close to MACRO limits.
  - Leptogenesis is automatic.
- susy hybrid inflation models are nicely related to susy GUTs. WMAP3 appears to prefer non-minimal Kähler potential. Nice properties are retained.

- Much rests on

Planck (07) !

Precision Cosmology needed to distinguish between models.

Summary

- Neutrinos have been 'around' for a long time (compare quarks) and they remain very elusive:

  Masses m_{ν_i} ?

  Third mixing angle θ_{13} ?

  CP violating phase δ ?

  Majorana or Dirac ?

- Rare decays: $\mu \to e\gamma$, $\mu \to eee$, ...

  These will provide tests for models of ν masses/mixings.

- Cosmic Neutrino Background?
- KK particles in warped models may be found at LHC.
- Low energy tests of leptogenesis?
- Seesaw at LHC?
- Precision Cosmology will test inflation models (Planck '07).